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Study on replication of a nonlinear dynamical system's trajectory using a machine learning technique

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Abstract. Nonlinear system exhibits various solution orbits depending on varying parameters. It is important to detect the system's behavior. In some cases, however, the mathematical modeling of the dynamic is completely unknown. By using a recent advance in the Machine Learning technique named Reservoir Computing, we replicate the solution orbits based only on data collected along with time evolution. We numerically confirm the effectiveness of Reservoir Computing in time series prediction.

1. Introduction
Recently, Machine Learning (ML) method, typically Deep Learning (DL) techniques have attracted intensive research interest in the nonlinear system [1-9]. By using limited data such as solutions of Ordinary Differential Equations (ODEs) or Partial Differential Equations (PDEs) as the input, the ML model is trained to estimate or replicate the evolution of the system. At the output, we then received an approximate solution's trajectory that is close to the real one. ML technique has shown great success in the task of nonlinear problems such as prediction of time-series evolution [3-5], chaos controlling [6, 7], or synchronization [8].

In principle, the study of a nonlinear dynamical system is often analyzed on a system that is known before. In many cases, nonlinear phenomena cannot be modeled in the accurate mathematical equation. Many researchers have proposed various ML techniques which are able to accomplish the desired purpose basing only on observed data. Among the ML methods, Reservoir Computing (RC) has been adopted to replicate the dynamic's trajectory. RC is a kind of Recurrent Neural Network (RNN) which is first employed by Jaeger et al. in [9]. The authors implemented such a model to learn the mechanism of biological brains. This method gained computational cost in the task of predicting chaotic time series over previous techniques. Further, it also showed the potential of engineering application in the communication field. In the same manner, Pathak et al. improved a procedure for calculating the Lyapunov exponent by using data observed in the Lorenz system and the Kuramoto-Sivashinsky system [4]. The proposed method was successful in computing the positive and zero Lyapunov exponents with great accuracy. However, it failed to predict the time-evolution for long iteration. Further, such a system is quite difficult to analyze higher dimensional dynamics.

Herein, we consider replicating the trajectory solution in a coupled piecewise-constant oscillators (PWCOs) by using the RC method. To the best of our knowledge, this is the first study of application of the ML technique on PWCOs. The detailed discussion of PWCOs is introduced in Ref. [10], the governing equation of which has a piecewise constant. Hence, it is relatively easy to obtain the rigorous