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Analysis And Optimization A Compliant Bistable Mechanism With Chained Beam Constrain Model

Van Duc Phan[†], Ngoc Tran Le[‡], Ngoc Dang Khoa Tran[‡*]

[†]Faculty of Automobile Technology, Van Lang University, Ho Chi Minh City, Vietnam.

[‡]Falcuty of mechanical engineering, Industrial University of Ho Chi Minh City, Ho Chi Minh City, Vietnam

[*]Corresponding Author Email: tranngocdangkhoa@iuh.edu.vn

ABSTRACT**:** This article presents a new novel to design a compliant bistable mechanism based on the optimization method combine with the numerical method. The chained beam constraint method (CBCM) is a powerful analysis method and predicts the bistable mechanism's nonlinear characteristics. The process discrete the beam into several small elements allow investigating the bending behavior of each element. In combination with the non dominated sorting genetic algorithm II (NSGA-II), the multi-objective optimization with the genetic algorithm is applied to design the bistable mechanism. The bistable mechanism comprises the inclined shape beam and the center mass employed to analyze and design. The difference between the numerical method and the finite element method is investigated. CBCM with many segments is examinated to predict the accuracy of the model.

KEYWORDS: Bistable mechanism, numerical method, CBCM, flexural beam.

INTRODUCTION

Compliant bistable mechanisms are employed in design many agencies in the MEMS devices [1], aerospace [2], latches [3], sensors [4], relays [5]. One benefit of these mechanisms is saved energy in the movement. The jointless structure increases the accuracy in the fabrication and reduces friction.

Figure 1 is the principle of the bistable mechanism. The device is comprised of four flexural beams and a central mass. The external force is applied to the center mass, which causes the mass to move to follow the y-direction. This mechanism has two stages in operation. The first stage is the initial position; when the mechanism works, the mechanism's center mass moves down and achieves the second stage. The center mass keeps this point with a stable equilibrium position without any energy to remain at this stage.

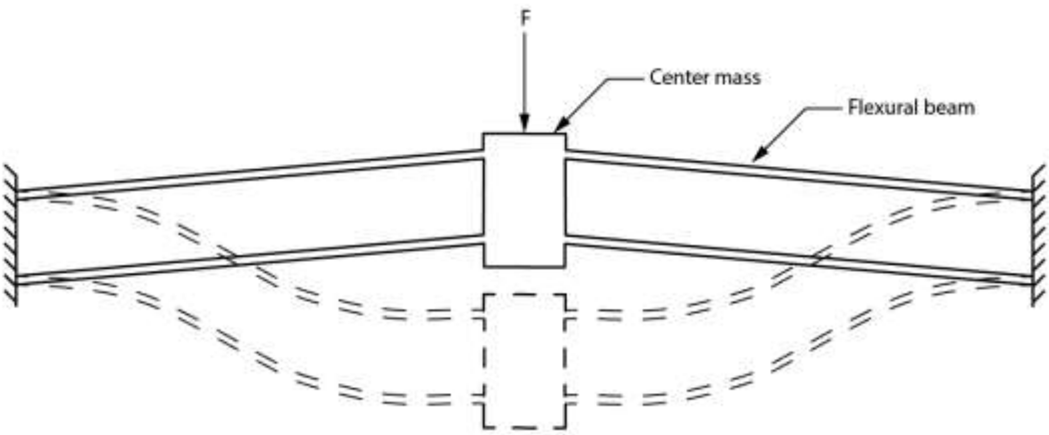

Figure 1. Original and deformed stage of bistable mechanism.

Typically, the rigid body of the quarter model of the bistable mechanism is illustrated in figure 2a. The mechanism includes two linkages and one spring. Both linkages are connected by a spring. One of the linkages has one end pinned, the other end of the linkage has an end roller that allows moving in the Y direction. When the roller moves down, the spring is compressed. Continue applying the force to the roller, the roller continues moving down and the spring released the push the roller jump to the second stable position. However, this model is very complicated in design and fabrication, especially in the micro-world. The compliant mechanism is employed to serve this mechanism. The compliant mechanism's replacement is shown in figure 2b; a flexible beam replaces the rigid linkages and the spring with high elastic bending and large deformations. The fixed end replaces the end pinned of the rigid linkage and The roller end remains. The mechanism becomes the statically indeterminate structures. The large deflection in compliant bistable mechanisms causes difficulty in predict and analyzing the behavior of the devices.

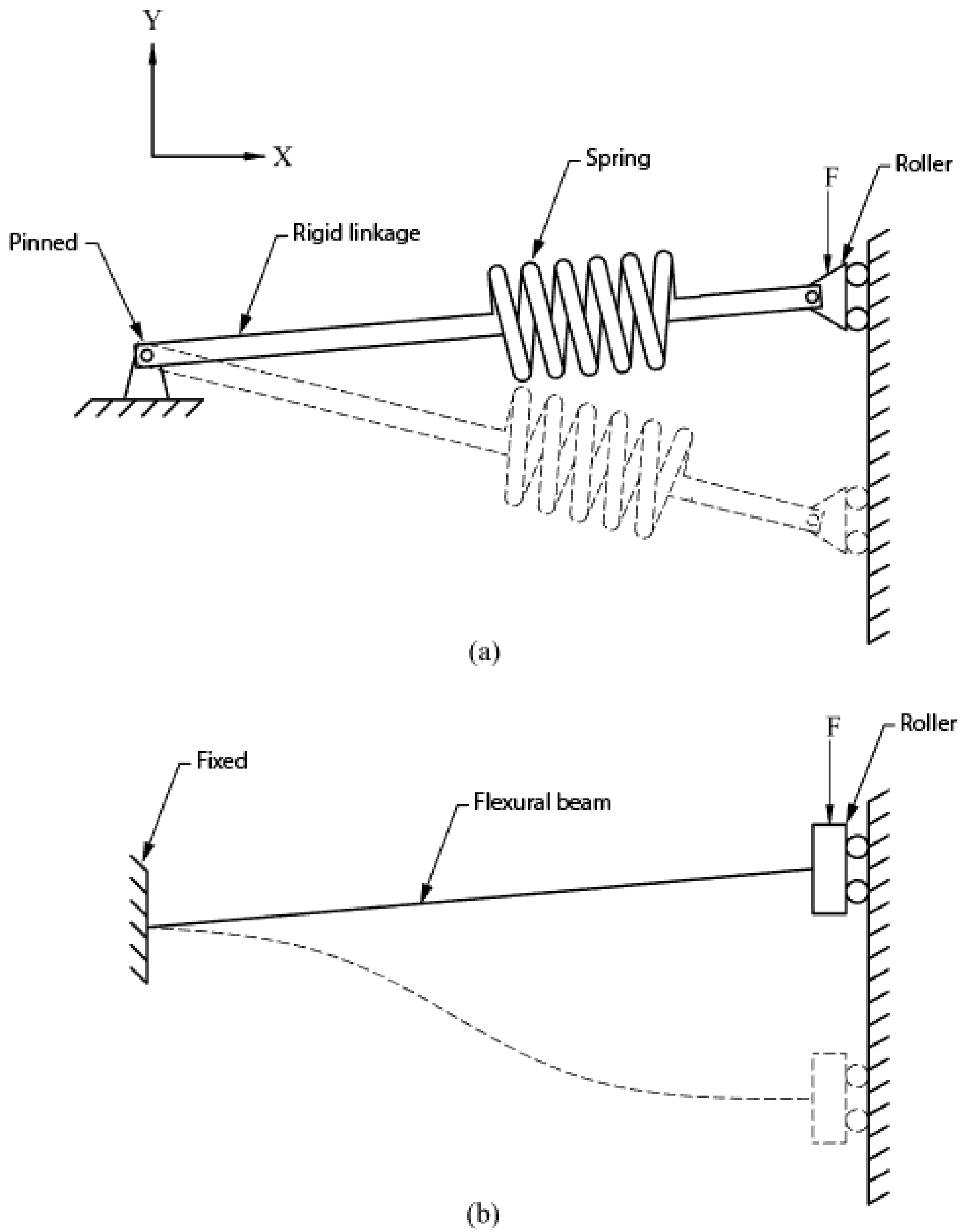

Figure 2. (a) Rigid body model and (b) compliant model of bistable mechanism

In order to analyze the characteristic of the bistable mechanism, the finite element method (FEM) is popularly utilized to solve. A report of the technique demonstrated by Jensen et al. [6], which assists the compliant bistable micromechanism design and quickly combines with the optimization method. The five segments bistable mechanism also uses the FEM method to analyze and create an equal force in backward and forward motion [7]. Chevron type bistable mechanism investigated based on FEM helps quickly acquire the structures' buckling behavior [8]. In contrast, the method requires an amount of time to process the analysis of the large deflection.

Several researchers have been introduced numerical methods for the analysis of the compliant bistable mechanism [9]. Almost all the methods are developed based on the Bernoulli-Euler beam theory [10]. Kimball and Tsai exploit the elliptic integral solution [11] to predict the compliant bistable mechanism's behavior but solve the numerical equation is very sophisticated. An improvement of the elliptical integral solution demonstrated by Holst et al. [12] helps analyze the large deflection of fixed end beams. A solving method with the Guass-Chebyshev quadrature equation is introduced by Saxena and Kramer [13] to help diagnose the large deflection of beams. Hussein et al. [14] improve the buckling equation to model the snapping force and stress in the bistable mechanism's operation. The high modes model improves the accuracy to predict the characteristic of the structure. In combination with Timoshenko beam theory, Chen and Ma [15] develop a numerical method that considers the shear factor in analyzing the bistable mechanism's behavior. The chain beam constraint model (CBCM) divides the bistable beam into many segments and applies the beam constraint model for each beam to investigate the nonlinear characteristic in the compliant mechanism [16]. Some kinds of research employ the CBCM to predict the behavior of the compliant devices are reported, such as accelerometer [17], the thermal actuator [18], gripper [19].

Many optimization methods are employed to design the compliant bistable mechanism. The combination of the numerical method and the optimization method to design the curved beam bistable mechanism is introduced by Hussein et al. [20]. Huang et al. [21] developed the optimization method for the symmetric and asymmetric bistable mechanism with many segments. An application of non dominated sorting genetic algorithm II (NSGA-II) for optimal the bistable structure also demonstrated [22] help design with high precious.

The work of this paper applies the CBCM method to analyze the compliant bistable mechanism with inclined slender beams and combination with the NSGA-II to design the structure. The comparison of calculus and FEM results are executed to evaluate the accuracy of the method.

MODELLING

Because of the symmetry of the model, a quarter model of the bistable mechanism is considered in the numerical analysis method. The center mass is modeled as a rigid body. Due to the quarter model is investigated, the center mass is display as a roller boundary condition. The chain beam constraint model (CBCM) separates the flexural beam into some segments is shown in figure 3a. The beam is divided into n segments. Each segment has the design parameters consist of the length L, the width w, the thickness t and the incline angle θ of the beam follow the figure 3b. δ is the displacement of the center mass follow the global coordinate. The beam is uniform in material properties, with the same Young's Modulus and Poisson's ratio. The subscript i refer to the ith segment of the flexural beam.

Figure 4a demonstrates the free body diagram of the first segment of the flexural beam. The XY coordinate is roled as the global coordinate, X_1Y_1 is the local coordinate of the first segment. The forces and moment place in the free end of the beam. When the beam deformed, the dash line beam shows the deflection of the beam. With ΔX_1 and ΔY_1 is the displacement of the beam with the local coordinate. The α_1 is the slope of the first segment. Figure 4b illustrates the free body diagram of the other segments of the beam. In these segments, one end of the beam is put the reaction forces and reaction moment of the previous beam and the other end of the beam is analyzed as the first beam. The moment at the free end is on the right-hand side equal to the moment at the end on the left-hand side. The free-body diagram of the final segment of the beam is presented in figure 4c. The final segment connects to the center mass, which moves in the Y direction. In the free end of the segment, the reaction forces P_i, F_i and moment M_i are applied from the previous beam, the other end connects with the roller and the analysis of the roller is shown in the figure. In each segment, six variables F_i, P_i, α_i, M_i, ΔX_i, ΔY_i, are the unknown value.

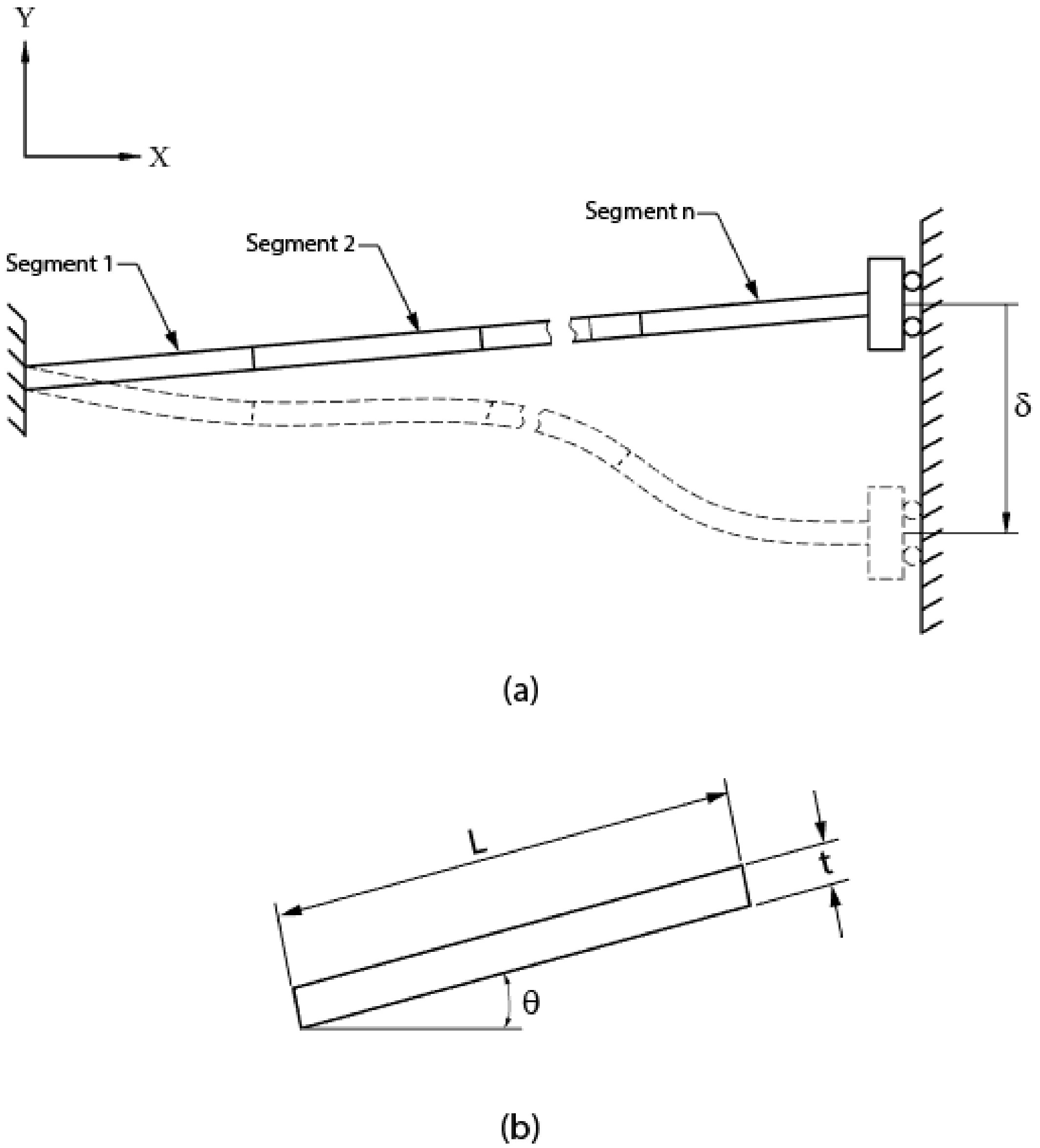

Figure 3. (a) Discrete beam into several segments and (b) the parameter of a segment.

The beam constraint model (BCM) applied to each segment is written follow [23]:

$$\begin{bmatrix} f_i \\ m_i \end{bmatrix} = \begin{bmatrix} 12 & -6 \\ -6 & 4 \end{bmatrix} \begin{bmatrix} \delta Y_i \\ \alpha_i \end{bmatrix} + p_i \begin{bmatrix} \frac{5}{6} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{2}{15} \end{bmatrix} \begin{bmatrix} \delta Y_i \\ \alpha_i \end{bmatrix} + p_i^2 \begin{bmatrix} -\frac{1}{700} & \frac{1}{1400} \\ \frac{1}{1400} & -\frac{11}{6300} \end{bmatrix} \begin{bmatrix} \delta Y_i \\ \alpha_i \end{bmatrix} \quad (1)$$

$$\delta X_i = \frac{t_i^2 p_i}{12 L_i^2} - \frac{1}{2} \begin{bmatrix} \delta Y_i & \alpha_i \end{bmatrix} \begin{bmatrix} \frac{5}{6} & -\frac{1}{10} \\ -\frac{1}{10} & \frac{2}{15} \end{bmatrix} \begin{bmatrix} \delta Y_i \\ \alpha_i \end{bmatrix} - p_i \begin{bmatrix} \delta Y_i & \alpha_i \end{bmatrix} \begin{bmatrix} -\frac{1}{700} & \frac{1}{1400} \\ \frac{1}{1400} & -\frac{11}{6300} \end{bmatrix} \begin{bmatrix} \delta Y_i \\ \alpha_i \end{bmatrix} \quad (2)$$

In this equation, all the variables are normalize and given the equations :

$$m_i = \frac{M_i L_i}{EI_i}, f_i = \frac{F_i L_i^2}{EI_i}, p_i = \frac{F_i L_i^2}{EI_i}, \delta Y_i = \frac{\Delta Y_i}{L_y}, \delta X_i = \frac{\Delta X_i}{L_x} \quad (3)$$

Based on three BCM equations, the equation for the whole flexural beam is **3*i** equations. The other equations are the relation between the reaction force and the reaction moment of the previous segment and the current segment. The equations are given:

$$P_{i-1} = [P_i \quad F_i] \begin{bmatrix} \cos(\theta_i - \theta_{i-1} + \alpha_{i-1}) \\ -\sin(\theta_i - \theta_{i-1} + \alpha_{i-1}) \end{bmatrix} \quad (4)$$

$$F_{i-1} = [P_i \quad F_i] \begin{bmatrix} \sin(\theta_i - \theta_{i-1} + \alpha_{i-1}) \\ \cos(\theta_i - \theta_{i-1} + \alpha_{i-1}) \end{bmatrix} \quad (5)$$

$$M_{i-1} = M'_{i-1} = M_i + [P_i \quad F_i] \begin{bmatrix} -\Delta Y_i \\ (L_i + \Delta X_i) \end{bmatrix} \quad (6)$$

For each connection of two segments, three equations are indicated. Then for all segments of the beam, 3*(i-1) equations are defined. The final equation involves the geometry of the beam; they include the geometry in the x and y direction follow the global coordinate and the total slope of the beam.

$$L_x = \sum_{i=1}^{n} \left([(L_i + \Delta X_i) \quad \Delta Y_i] \begin{bmatrix} \cos(\theta_i + \alpha_{i-1}) \\ -\sin(\theta_i + \alpha_{i-1}) \end{bmatrix} \right) \quad (7)$$

$$L_y + \delta = \sum_{i=1}^{n} \left([(L_i + \Delta X_i) \quad \Delta Y_i] \begin{bmatrix} \sin(\theta_i + \alpha_{i-1}) \\ \cos(\theta_i + \alpha_{i-1}) \end{bmatrix} \right) \quad (8)$$

$$\sum_{i=1}^{n} \alpha_i = 0 \quad (9)$$

All the equations of the flexural beam are indicated and Matlab program execute to solve the equation and find the results.

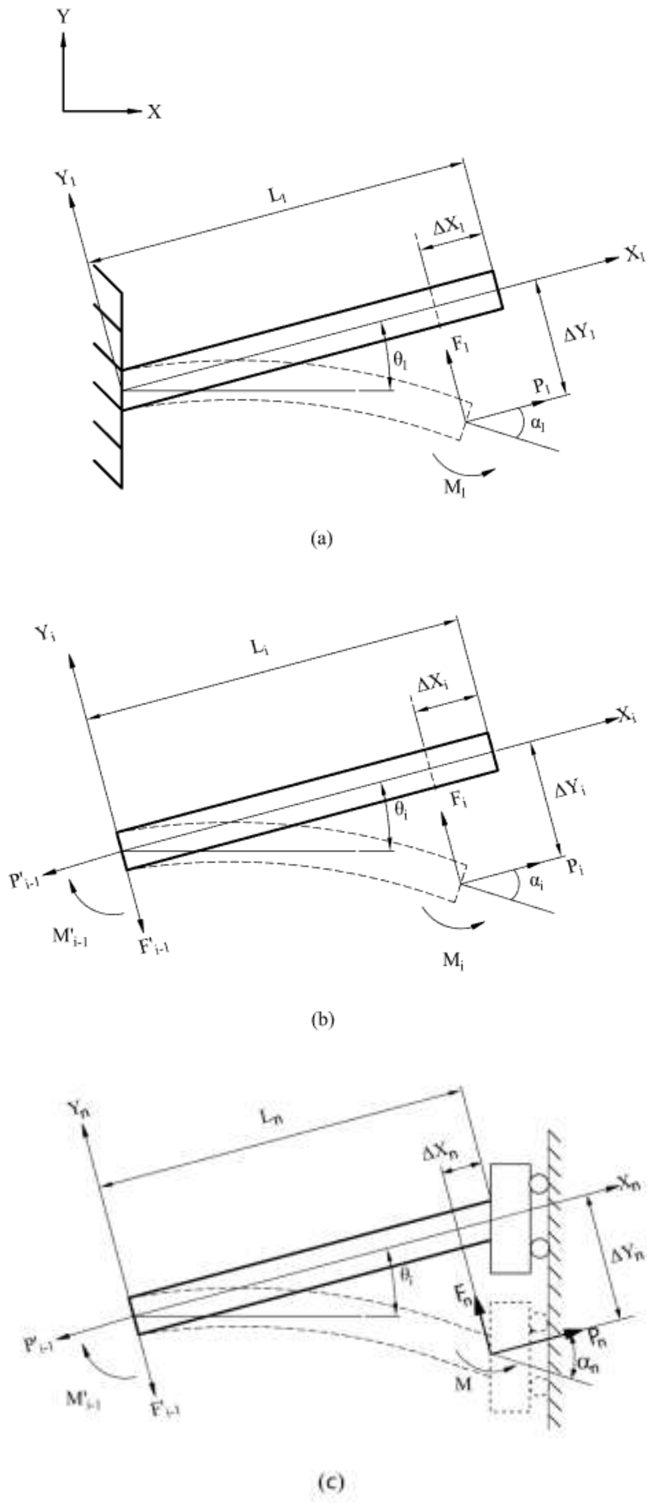

Figure 4. Free body diagram of the each segment of the flexural beam.

OPTIMAL DESIGN

The optimal design of a compliant bistable mechanism is implemented by the NSGA-II method. The calculus of the nonlinear behavior of the bistable mechanism is based on the CBCM method. Figure 5 shows the process of optimal design. The design parameters are installed in the Matlab file. When the program is run, random parameters are created and put into the CBCM file. After the program solves the equations in CBCM, a characteristic of force-displacement relation is achieved. The program continues to find the maximum force and minimum force of the mechanism. The values compare with the objective functions and find the individual population. Then the genetic algorithm starts to generate the community and find the final solution.

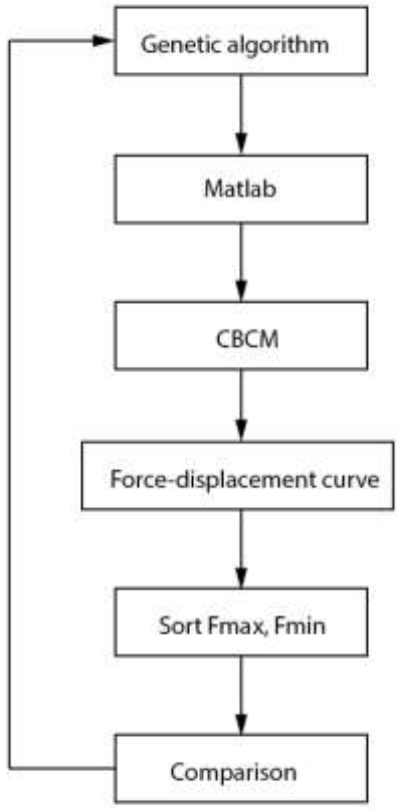

Figure 5. Process of the optimal method

An optimal design of the compliant bistable mechanism is implemented with CBCM, the beam is divided into 5 segments. The lower bound and upper bound of the parameter is given in table 1. The polyoxymethylene (POM) is carried on the material of the device. The properties of the material are 2.1 Gpa in Young's modulus and Poisson's ratio is 0.6. The objective function is the maximum value and minimum value force are 1600 (mN) and -1250 (mN).

The objective functions of the mechanism are given:

Min |Fmax-1600|

Min |Fmin-1300|

Table 1. Variable parameters for the bistable model.

Variables	Lower bound	Upper bound
L_1 (mm)	3	10
L_2 (mm)	3	8
L_3 (mm)	20	70
L_4 (mm)	3	8
L_5 (mm)	3	10
θ_1 (degree)	-5	6
θ_2 (degree)	1	10
θ_3 (degree)	1	10
θ_4 (degree)	1	10
θ_5 (degree)	-5	6

The generation is taken as 40 and the population of each generation is 20. Figure 6 show the result of 20th, 30th and 40th generation of optimization. The two coordinates present the values of the objective function. In the generation 20th, many points are far the target of objective functions. At the 40th generation, the optimal solution is selected the value of the point which marked in the figure. Figure 7 displays the behavior of the optimum design, the value of maximum force is 1608 mN and minimum force is -1257 mN, which the errors compare with the target value are around 1%. The value of the design parameters are showed in table 2.

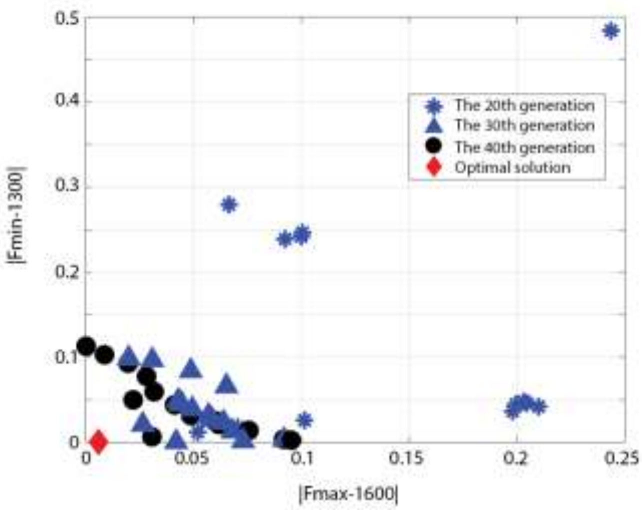

Figure 6. The generation of optimization

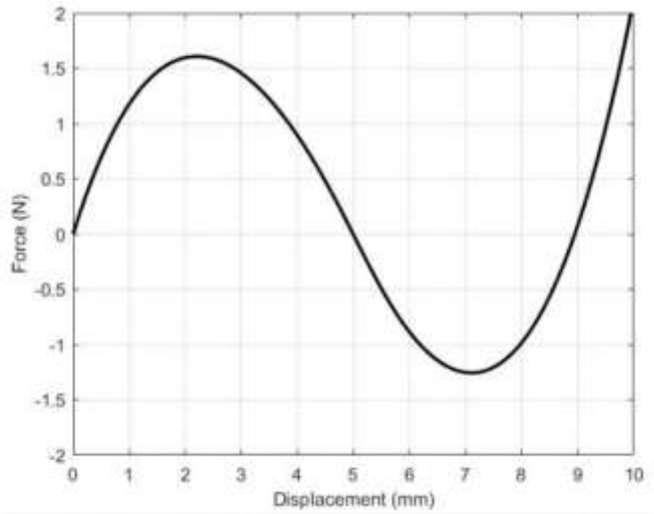

Figure 7. The optimal solution of NSGA-II

Table 2. Results of optimal solution.

Variables	Values
L_1 (mm)	5.2
L_2 (mm)	4.8
L_3 (mm)	40
L_4 (mm)	4.7
L_5 (mm)	5.2
θ_1 (degree)	0
θ_2 (degree)	5.2
θ_5 (degree)	5.2
θ_5 (degree)	5.2
θ_5 (degree)	0

RESULT AND DICUSSION

The finite element method is carried out to verify the result of the optimization. Figure 8a shows the meshing of the bistable mechanism, which the parameters are performed in table 2. The figure 8b illustrates the detail meshing of the one flexural beam. The Abaqus sofware is executed to analysis the behavior of the model. The material properties are same with the optimum process.

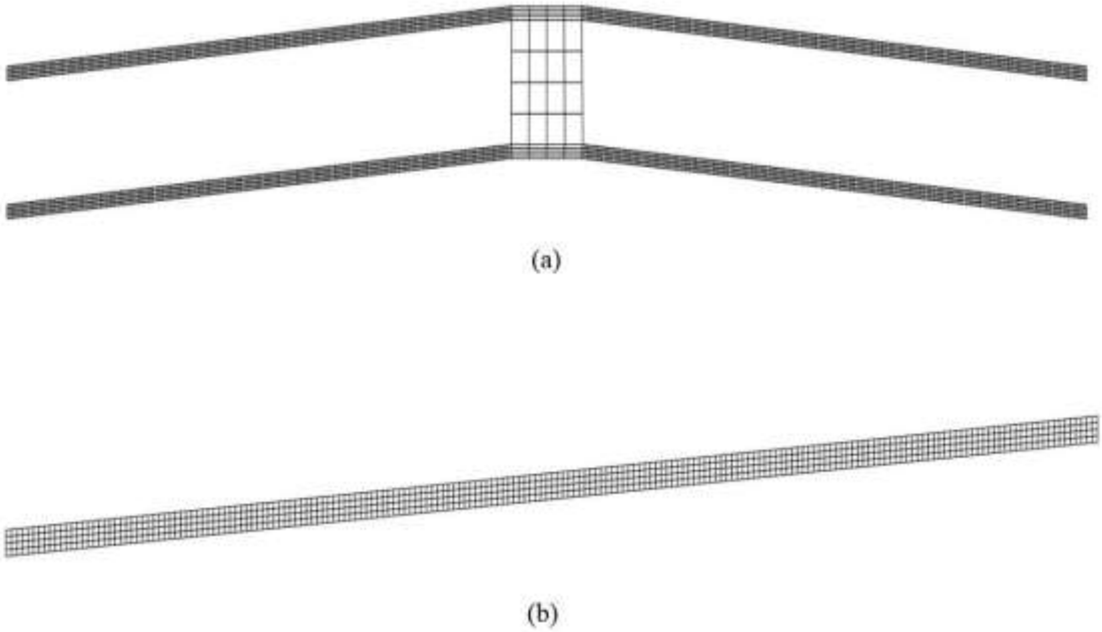

Figure 8. (a) Meshing model of the bistable mechanism, (b) meshing of the flexural beam.

A comparison of the FEM method with the CBCM is shown in figure 9. The FEM gives the result close to the CBCM method, with the error is 10%. The CBCM with many segments is also presented in the figure, the differences of these results are minimal. The values of the result is shown in table 3. The processing time also showed in the table. In FEM, the costing time includes the drawing, meshing and processing analysis. When change any design parameters, the FEM should be draw the new model, meshing and set up the new boundary conditions and do simulation again. Therefore, the FEM process takes long time to implement compare with the CBCM method.

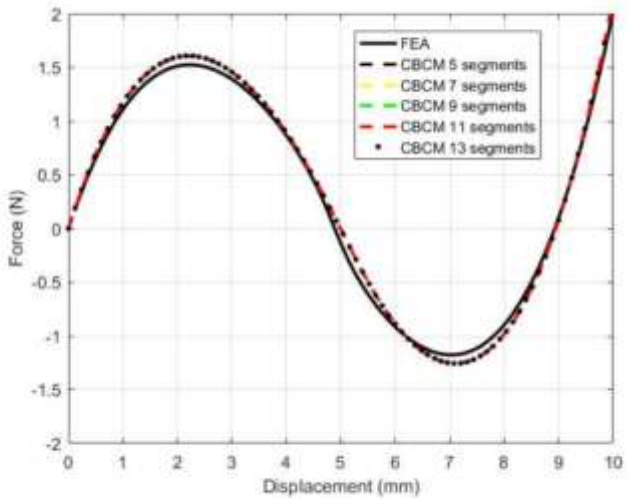

Figure 9. FEA and CBCM result

Table 3. Comparision between CBCM and FEM method.

Method	**Fmax (N)**	**Fmin (N)**	**Processing time (sec)**
FEM	1.524	-1.17	300
CBCM 5 segments	1.608	-1.257	25
CBCM 7 segments	1.618	-1.267	37
CBCM 9 segments	1.608	-1.267	47
CBCM 11 segments	1.628	-1.268	56
CBCM 13 segments	1.628	-1.268	73

CONCLUSION

The new method to design a compliant bistable mechanism is presented by combining the numerical method to predict the nonlinear behavior of the model and the optimal method. The analysis of the compliant bistable mechanism is implemented with the numerical method based on CBCM. The compliant bistable mechanism comprised of four straight beams is employed to analyze. Matlab is employed to solve the equations. The numerical method has the advantage in the predict the nonlinear behavior in save time. A simple model of the compliant bistable mechanism is investigated, which formed on the inclined guided beams and a central mass. The optimization with two objective functions serves for the model involve the maximum force and minimum force. After executing the optimal method, the CBCM method's results are verified to the FEM model which the errors are minimal, below 10%. The computation with a high discrete segment acquires the precious work, but it cost more time to analyze. CBCM assists design better than the FEM design.

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